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Theoretical Model for $\pi + \mathcal{N} \to \mathcal{N} + \gamma$.

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Some data are now available on the cross-section of $\pi^- + p \to n + \eta$ (1,2). The charge symmetric reaction $\pi^+ + n \to p + \eta$ has been observed on a deuteron target (3). The data indicate a very fast rise of the cross-section soon after the threshold, followed by a slow decrease. We have developed a model for this reaction, based on simple assumptions, which provides a reasonable fit to the data.

The following considerations have led us to the choice of a particular theoretical model. First, from invariance considerations, in the crossed reaction $\pi + \eta \to \mathcal{N} + \overline{\mathcal{N}}$ no important low-mass singularities are expected: the lowest-mass intermediate state is a 3π state; intermediate ρ or ω states do not contribute. Second, the available data (2) show that the cross-section does not apparently feel the presence of the so-called «third» nucleon-pion resonance (at ~ 900 MeV incident pion kinetic energy).

In our model we assume that the amplitude is dominated by the Born terms and by a term arising from formation and decay of the «second» nucleon-pion resonance ($\overline{\mathcal{N}}^*_{\frac{3}{2}}$ with a mass $M = 1512$ MeV). The Born terms are derived from the couplings $g\bar{\psi}i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\varphi}\psi$ for the π-nucleon vertex and $g_\eta\bar{\psi}i\gamma_5\eta\psi$ for the η-nucleon vertex, where η is the field describing η^0. The resonant amplitude is derived from a coupling $(G/\mu)F\bar{\psi}i\gamma_5\boldsymbol{\tau}\partial_\lambda\boldsymbol{\varphi}\psi_\lambda$ for the $\mathcal{N}^*\mathcal{N}\pi$ vertex and a coupling $(G_\eta/\mu)F\bar{\psi}i\gamma_5\partial_\lambda\eta\psi_\lambda$ for the $\mathcal{N}^*\mathcal{N}\eta$ vertex. In these expressions μ is the pion mass, F is a form factor, and ψ_λ denotes the spin-$\frac{3}{2}$ isobar, described as a Rarita-Schwinger field (4). In ψ_λ the index λ is a four-vector index, and the indication of the spinor index is omitted. The field ψ_λ satisfies $(\gamma\partial + M)\psi_\lambda = 0$ and the subsidiary conditions $\gamma_\lambda\psi_\lambda = 0$ and $\partial_\lambda\psi_\lambda = 0$.

(1) R. J. Cence, V. Z. Peterson, V. J. Stenger, C. B. Chiu, R. D. Eaudi, R. W. Kenney, B. J. Moyer, J. A. Poirier and W. B. Richards: to be published.

(2) F. Bulos, R. E. Lanou, A. E. Pifer, A. M. Shapiro, M. Widgoff, R. Panvini, L. Guerriero, G. Calvelli, G. A. Salandin, A. Tomasin, L. Ventura, C. Voci, F. Waldner, C. A. Bordner, A. E. Brenner, M. E. Law, E. E. Ronat, K. Strauch, J. J. Szymansky, P. Bastien, B. Brabson, Y. Eisenberg, B. T. Feld, V. K. Fischer, L. A. Pless, L. Rosensons and R. K. Yamanoto: *Phys. Rev. Lett.*, **13**, 487 (1964).

(3) T. Toohig, R. Kraemer, L. Madansky, M. Meer, A. Pevsner, C. Richardson, R. Strand and M. Block: *Proc. of the Intern. Conference on High-Energy Physics at CERN* (1962), p. 99.

(4) W. Rarita and J. Schwinger: *Phys. Rev.*, **60**, 61 (1941).

For the calculation of the intermediate $\mathcal{N}^*$ amplitude one needs the spin-$\frac{3}{2}$ projection operator $P_{\mu\nu}$ given by (5)

$$P_{\mu\nu} = \frac{-i\gamma\omega + M}{2M}\left[\delta_{\mu\gamma} - \frac{1}{3}\gamma_\mu\gamma_\nu + \frac{i}{3M}(\gamma_\mu\omega_\nu - \gamma_\nu\omega_\mu) + \frac{2}{3M^2}\omega_\mu\omega_\nu\right],$$

where ω is the $\mathcal{N}^*$ momentum. The resonant amplitude is given by

$$\frac{GG_\eta}{\mu^2}FF'\frac{2M}{\omega^2 + M^2}\overline{u}_f(p')\gamma_5 P_{\mu\nu}\gamma_5 u_i(p) q'_\mu q_\nu ,$$

where p, q and p', q' are the initial and final momenta of the nucleon and of the boson, respectively. To take into account the finite $\mathcal{N}^*$ width we shall substitute in the denominator $(\omega_0^2 - M^2)^2$ with $(\omega_0 + M)^2[(\omega_0 - M)^2 + \frac{1}{4}\Gamma^2]$, where Γ is the (energy-dependent) width of $\mathcal{N}^*$. For the form factor F we use tentatively $F(\boldsymbol{q}) = (1 + \boldsymbol{q}^2/Q^2)^{-1}$, where Q is expected to lie between $1 \div 3$ pion masses, as in similar cases (6). In the $\mathcal{N}^*$ rest system $2M\gamma_5 P_{\mu\nu}\gamma_5 q'_\mu q_\nu$ reduces to

$$\left(\frac{4}{3}\right) M|\boldsymbol{q}||\boldsymbol{q}'|\frac{1}{2}(1 - \gamma_4)\left(-\cos\theta + \frac{i}{2}\sin\theta\sigma_x\right),$$

where θ is the scattering angle and x is the direction normal to the reaction plane. This expression is essentially the $d_{\frac{3}{2}}$ projection operator: the resulting resonant amplitude consists of a nonspin-flip part proportional to $\frac{1}{3}(1 - 3\cos^2\theta)$ and a spin-flip part proportional to $-i\sin\theta\cos\theta$. The angular distribution is thus of the form $1 + 3\cos^2\theta$, as expected for a $d_{\frac{3}{2}}$ wave. The partial width for $\mathcal{N}^* \to \mathcal{N} + \pi$ ($n + \pi^0$ plus $p + \pi^-$) is given by

$$\Gamma_\pi = \left(1 + \frac{\boldsymbol{q}^2}{Q^2}\right)^{-2}\frac{1}{4\pi}\frac{G^2}{\mu^2}\frac{|\boldsymbol{q}|^3}{M}(p_0 - m), \tag{1}$$

where m is the nucleon mass, and similarly the partial width for $\mathcal{N}^* \to n + \eta$ by

$$\Gamma_\eta = \left(1 + \frac{\boldsymbol{q}'^2}{Q^2}\right)^{-2}\frac{1}{4\pi}\frac{1}{3}\frac{G_\eta^2}{\mu^2}\frac{|\boldsymbol{q}'|^3}{M}(p'_0 - m). \tag{2}$$

For the total width of $\mathcal{N}^*_{\frac{1}{2}}$ (1512) we take $\Gamma \sim 90$ MeV (7). We also put tentatively $\Gamma = \Gamma_\pi + \Gamma_\eta$. Inelastic channels may well be important (8) in the decay of $\mathcal{N}^*_{\frac{1}{2}}$ (1512), but it is hard to establish how far could large inelastic contributions modify the conclusions. Qualitatively one would expect the resonant amplitude to be depressed.

The expressions for the cross-section are given in the Appendix.

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(5) V. GLASER and B. JACKSIC: *Nuovo Cimento*, **5**, 1197 (1957).

(6) See for instance M. GELL-MANN and K. WATSON: *Ann. Rev. Nucl. Sci.*, **4**, 219 (1954).

(7) W. H. BARKAS and A. H. ROSENFELD: Lawrence Radiation Laboratory Report, UCRL-8030 (unpublished, 1963).

(8) J. A. HELLAND, J. J. DEVLIN, D. E. HAGGE, M. J. LONGO, B. J. MOYER and C. D. WOOD: *Proc. of the Intern. Conference on High-Energy Physics at CERN* (1962), p. 3.

The data represent by circles in Fig. 1 are obtained from those of ref. (2), where production of η^0 with subsequent $\eta^0\to 2\gamma$ was measured, by multiplying by a factor of 2.6, as suggested by BERTHELOT (9) to account for the fraction of η^0 decaying into 2γ (2γ branching ratio $=38\%$). The data represented by squares are obtained, still from the data of ref. (2), by multiplying instead by a factor of 3.1, derived from another possible choice for the branching ratio (2γ branching ratio $=32\%$), still consistent with the present confused experimental situation on these branching ratios (10). The two sets of data are meant to be representatives of possible behaviours of the cross-sections and are reported to stress on the magnitudes of the experimental errors.

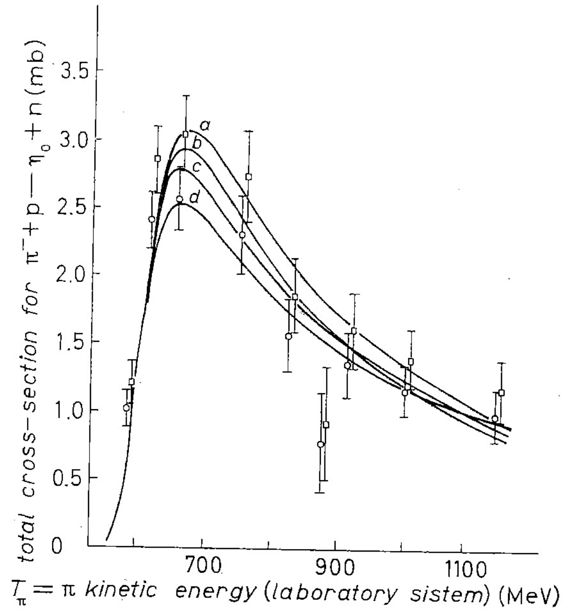

Fig. 1. – Total cross-section for $\pi+\mathcal{N}\to\mathcal{N}+\eta$. The data are those of ref. (2), corresponding to a $\eta^0\to 2\gamma$ branching ratio of 38 % (points represented by circles) or of 32 % (points represented by squares). The fits *a*), *b*), *c*), and *d*) illustrate different choices of theoretical parameters.

Possible fits to the data for the total cross-section are shown in Fig. 1. They are only meant to be illustrative and different fits may be easily obtained from the expressions reported in the Appendix. The curves denoted by *a*) and *b*) were obtained for $\Gamma=90$ MeV, $\Gamma_\eta/\Gamma_\pi=4\%$, $g_\eta=0.30$ and $Q=2.35m_\pi$ and $2.12m_\pi$, respectively. The curves denoted by *c*) and *d*) are for $\Gamma=70$ MeV, $Q=2.58m_\pi$, $g_\eta=0.35$ and $\Gamma_\eta/\Gamma_\pi=4\%$ and 3.5%, respectively. We recall that, in full unitary symmetry, g and g_η are related by $g_\eta=(1/\sqrt{3})\,g(4\alpha-1)$. The parameter α which fixes the ratio of F to D coupling in the unitary symmetric interaction of the meson octet to the baryon octet ($\alpha=0$ for pure D, $\alpha=1$ for pure F) was suggested to be around $\frac{1}{4}$ by MARTIN and WALI (11). Our values of g_η are consistent with α around $\frac{1}{4}$, for which g_η is very small. The angular distributions are reported in Fig. 2 and 3. Our examples show how the distributions may in some cases be approximately isotropic, as a result of the coherence of the Born and resonant amplitudes.

The data can also be interpreted only in terms of dominant $s_{\frac{1}{2}}$ production with some higher l admixtures at high energy (2). The resonant contribution

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(9) A. BERTHELOT: *Proc. of the Sienna Conference on Elementary Particles* (1963), p. 60.

(10) The ratio of 38 % derives from assuming ($\eta\to$neutrals)/($\eta\to$charged) $\simeq 3$

$$(\eta\to 2\gamma)/(\eta\to 3\pi^0)\simeq 1\,,\qquad (\eta\to 2\pi\gamma)/(\eta\to\pi^+\pi^-\pi^0)\simeq 0.2\,.$$

The ratio of 32 % comes from ($\eta\to$neutrals)/($\eta\to$charged) $\simeq 2.5$,

$$(\eta\to 2\gamma)/(\eta\to 3\pi^0)\simeq 0.8\,,\qquad (\eta\to 2\pi\gamma)/(\eta\to\pi^+\pi^-\pi^0)\simeq 0.26\,.$$

In both cases no $\pi^0\gamma\gamma$ is assumed to contribute. The η branching ratios are discussed in detoil by C. BACCI, C. MENCUCCINI, G. PENSO, R. QUERZOLI, G. SALVINI, G. SILVESTRINI and A. WATTENBERG, LNF. 64/6, (1964); see also the paper in *Phys. Rev. Lett.*, **1**, 37 (1963), by the some authors.

(11) A. W. MARTIN and K. C. WALI: Argonne National Laboratory preprint (1962).

from $\mathcal{N}^*$ (1512) would then be negligible in comparison with the large S state and some mechanism would have to be invented to interpret the fast rise and subsequent decrease of the total cross-section. Finally, the apparent insensitivity of the cross-section to the presence of the third resonance in the region around

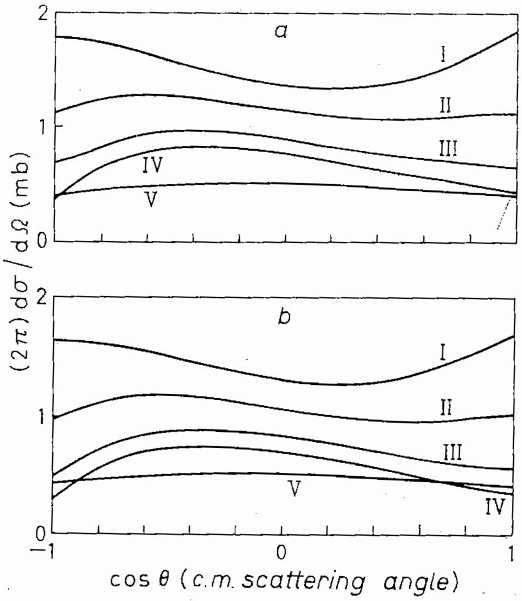

Fig. 2. – Predicted differential cross-sections corresponding to the fits *a*), *b*), to the total cross-section:
curve I - $T_\pi = 691$; $W = 1570$;
curve II - $T_\pi = 793$, $W = 1630$;
curve III - $T_\pi = 917$, $W = 1700$;
curve IV - $T_\pi = 1008$, $W = 1750$;
curve V - $T_\pi = 592$, $W = 1510$.

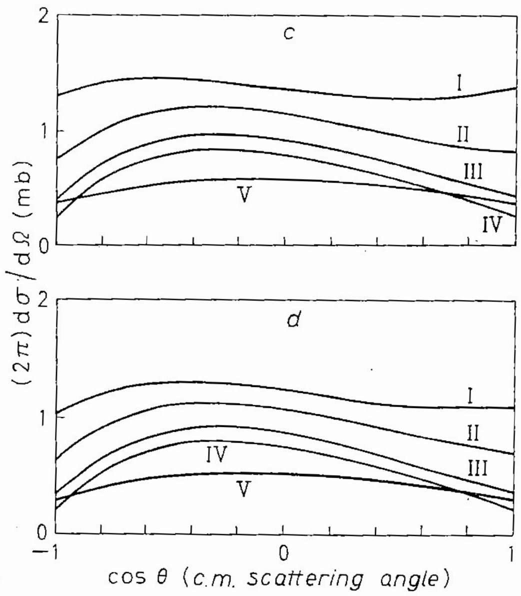

Fig. 3. – Predicted differential cross-sections corresponding to the fits *c*), *d*) to the total cross-section:
curve I′ - $T_\pi = 691$, $W = 1570$;
curve I″ - $T_\pi = 707$, $W = 1580$;
curve II - $T_\pi = 793$, $W = 1630$;
curve III - $T_\pi = 917$, $W = 1700$;
curve IV - $T_\pi = 1008$, $W = 1750$;
curve V - $T_\pi = 592$, $W = 1510$.

900 MeV incident pion kinetic energy is consistent with assigning $\mathcal{N}^*_{\frac{1}{2}}$ (1688) to an octet [12] (Regge recurrence of the baryon octet), for which, as for the baryon octet, the F/D mixing ratio is such as to give a very small coupling of the resonant state to $\mathcal{N}+\eta$. Alternatively, the insensitivity may simply reflect a large centrifugal barrier for decay of the $\frac{5}{2}^+$ state.

It will be interesting to find out which of all the above alternatives are true, when better data will be available.

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We thank Prof. Moyer and Dr. Guerriero for kindly sending us their preliminary results and Doctors Eisenberg, Brenner, Lanou and Pevsner for discussions on the data.

([12]) S. L. Glashow and A. Rosenfeld: *Phys. Rev. Lett.*, **10**, 192 (1963).

Appendix

Our result for the differential cross-section is

$$\frac{\mathrm{d}\sigma}{\mathrm{d}(\cos\theta)}=\frac{M^2}{36\pi}\frac{G^2G_\eta^2}{\mu^4}\frac{|\boldsymbol{q}||\boldsymbol{q}'|^3}{\omega_0^2}\frac{(p_0-m)(p_0'-m)}{(\omega_0+M)^2}\frac{1+3\cos^2\theta}{(\omega_0-M)^2+\frac{1}{4}\Gamma^2}F^2(|\boldsymbol{q}|)F^2(|\boldsymbol{q}'|)+$$
$$+\frac{g^2g_\eta^2}{16\pi}\frac{(p_0+m)(p_0'+m)}{\omega_0^2}\frac{|\boldsymbol{q}'|}{|\boldsymbol{q}|}\left(\frac{1}{\omega_0^2-m^2}+\frac{1}{2p_0'q_0-\mu^2+2|\boldsymbol{q}||\boldsymbol{q}'|\cos\theta}\right)^2\cdot$$
$$\cdot(B_0^2+2B_0B_1\cos\theta+B_1^2)\pm\frac{GG_\eta gg_\eta}{12\pi}\frac{|\boldsymbol{q}||\boldsymbol{q}'|^3M}{\mu^2\omega_0^2(\omega_0+M)}\frac{-B_0+2B_1\cos\theta+3B_0\cos^2\theta}{[(\omega_0-M)^2+\frac{1}{4}\Gamma^2]^{\frac{1}{2}}}\cdot$$
$$\cdot\left(\frac{1}{\omega_0^2-m^2}+\frac{1}{2p_0'q_0-\mu^2+2|\boldsymbol{q}||\boldsymbol{q}'|\cos\theta}\right)F(|\boldsymbol{q}|)F(|\boldsymbol{q}'|),$$

M, μ, m are respectively, the $\mathcal{N}^*$, π, $\mathcal{N}$ masses, ω_0 is the total c.m. energy, $\boldsymbol{q}$, $\boldsymbol{q}'$ are the c.m. initial and final boson momenta, q_0, q_0' are the corresponding c.m. energies and p_0, p_0' are the c.m. energies of the nucleons. Also

$$B_0=\omega_0-m\,,$$
$$B_1=|\boldsymbol{q}||\boldsymbol{q}'|\frac{\omega_0+m}{(p_0+m)(p_0'+m)}\,.$$

The total cross-section is given by

$$\sigma=\frac{M^2}{9\pi\omega_0^2}F^2(|\boldsymbol{q}|)F^2(|\boldsymbol{q}'|)\frac{G^2G_\eta^2}{\mu^4}\frac{|\boldsymbol{q}||\boldsymbol{q}'|^3(p_0-m)(p_0'-m)}{(\omega_0+M)^2[(\omega-M)^2+\frac{1}{4}\Gamma^2]}+$$
$$+\frac{g^2g_\eta^2(p_0+m)(p_0'+m)}{16\pi\omega_0^2}\frac{|\boldsymbol{q}'|}{|\boldsymbol{q}|}\left\{\frac{2(B_0^2+B_1^2)}{(\omega_0^2-m^2)^2}+\frac{1}{(\omega_0^2-m^2)|\boldsymbol{q}||\boldsymbol{q}'|}\cdot\right.$$
$$\cdot\left[4B_0B_1+\left(B_0^2+B_1^2-\frac{2p_0'q_0-\mu^2}{|\boldsymbol{q}||\boldsymbol{q}'|}B_0B_1\right)\log\frac{2p_0'q_0-\mu^2+2|\boldsymbol{q}||\boldsymbol{q}'|}{2p_0'q_0-\mu^2-2|\boldsymbol{q}||\boldsymbol{q}'|}\right]+$$
$$\left.+\frac{B_0B_1}{2|q|^2|\boldsymbol{q}'|^2}\log\frac{2p_0'q_0-\mu^2+2|\boldsymbol{q}||\boldsymbol{q}'|}{2p_0'q_0-\mu^2-2|\boldsymbol{q}||\boldsymbol{q}'|}+2\frac{(B_0^2+B_1^2)|\boldsymbol{q}||\boldsymbol{q}'|-(2p_0'q_0-\mu^2)B_0B_1}{|\boldsymbol{q}||\boldsymbol{q}'|[(2p_0'q_0-\mu^2)^2-4|\boldsymbol{q}|^2|\boldsymbol{q}'|^2]}\right\}+$$
$$+\frac{GG_\eta gg_\eta|\boldsymbol{q}'|^3|\boldsymbol{q}|MF(|\boldsymbol{q}|)F(|\boldsymbol{q}'|)}{12\pi\mu^2\omega_0^2(\omega_0+M)[(\omega_0-M)^2+\frac{1}{4}\Gamma^2]^{\frac{1}{2}}}\left\{\frac{1}{2|\boldsymbol{q}||\boldsymbol{q}'|}\left[-4B_1+\frac{3B_0}{|\boldsymbol{q}||\boldsymbol{q}'|}(2p_0'q_0-\mu^2)+\right.\right.$$
$$\left.\left.+\left(B_0+\frac{B_1}{|\boldsymbol{q}||\boldsymbol{q}'|}(2p_0'q_0-\mu^2)-\frac{3B_0}{4|\boldsymbol{q}|^2|\boldsymbol{q}'|^2}\right)\log\frac{2p_0'q_0-\mu^2+2|\boldsymbol{q}||\boldsymbol{q}'|}{2p_0'q_0-\mu^2-2|\boldsymbol{q}||\boldsymbol{q}'|}\right]\right\}.$$